“Is wake swirl and wake expansion important for rotor design?”

Or: Is the standard BEM model good enough for future’s wind turbines?

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Contents:

- *What is the BEM model to wind turbines?*
- *What does the original BEM contain?*
- *What does the original BEM not contain? – and why?*
- *Closing the gap …*
- *Corrected BEM results and benchmarks*
- *Perspectives:*
  - *Corrected BEM: “Nice to have” or “need to have”?*
  - *Closing more gaps.*

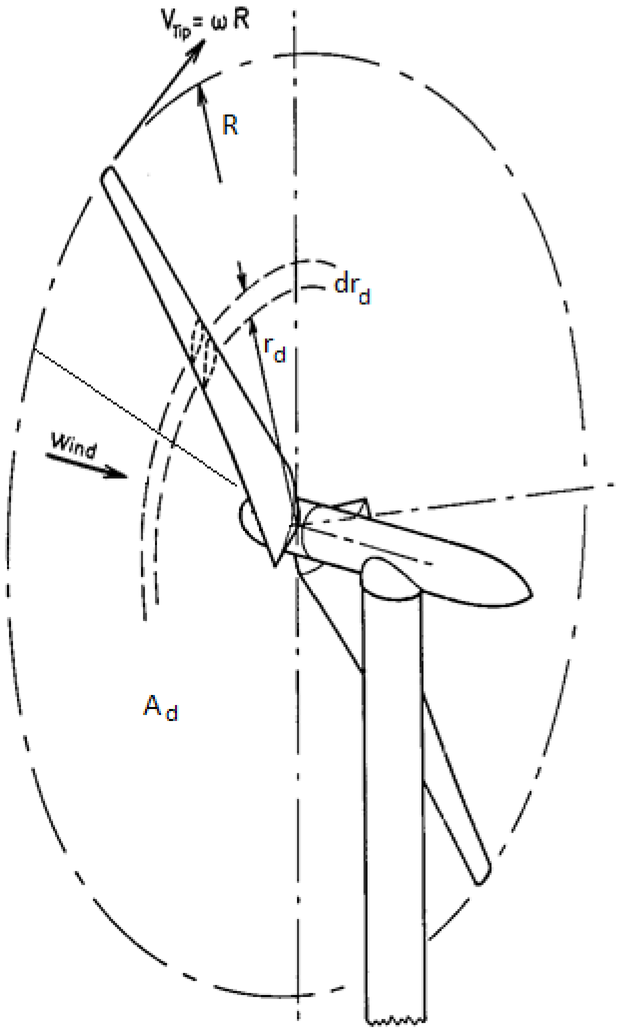
BEM stands for "Blade Element Momentum". It is the core module used by the entire industry as the aerodynamic work-horse in all wind turbine simulators:

- Design/optimization
- Evaluation
- Certification
- …

The BEM model is simple and fast, yet quite accurate !!

The founding "fathers" of BEM:

- Rankine (1865)
- Froude (1889)
- Joukowski (1918)
- Betz (1920)
- Glauert (1926, **1935**)



Assumptions/approximations:

- Aligned uniform inflow
- Axi-symmetry (no tiploss)
- No wake stall
- Annular element independence
- Simplified momentum conservation
  - Angular
  - Axial
- (In-plane disk/rotor)

What does the original BEM contain?

Axial momentum conservation	$F_x + \cancel{\Delta F_s} = \rho A_\infty U_\infty^2 - (\rho A_w U_w^2 + \cancel{A_w \Delta p_w})$
Angular momentum conservation	$\cancel{r} A_d U_d 2U_{\theta d} = \cancel{r_w} A_w U_w U_{\theta w}$
Mass conservation upstream	$A_\infty U_\infty = A_d U_d$
Mass conservation downstream	$A_d U_d = A_w U_w$
Mechanical energy conservation upstream (Bernouilli)	$\frac{1}{2}\rho U_\infty^2 = \frac{1}{2}\rho U_d^2 + \frac{1}{2}\rho U_{rd}^2 + p_{d+}$
Mechanical energy conservation downstream (Bernouilli)	$\frac{1}{2}\rho\left(U_d^2 + (2U_{\theta d})^2 + U_{rd}^2\right) + p_{d-} = \frac{1}{2}\rho\left(U_w^2 + U_{\theta w}^2\right) + \Delta p_w$
Disk axial forcing	$F_x = A_d(p_{d+} - p_{d-})$
Disk angular forcing	$F_\theta = \rho A_d U_d 2U_{\theta d}$
Geometry relation between disk forces	$\frac{F_x}{F_\theta} = \frac{\lambda U_\infty \frac{r}{R} + U_{\theta d}}{U_d}$

The red terms were purposefully left out in the original BEM equations for 2 reasons:

- Assumed negligibly small (and hard to compute …)
- Would cause mathematical coupling between stream tubes.

*What does the original BEM **not** contain? – and why?*

Axial projection of force
Interaction between
adjacent stream tubes

Radii of stream tubes
In fully developed (far)
wake

Far-wake pressure drop
Due to swirl

$$F_x + \Delta F_s = \rho A_\infty U_\infty^2 - (\rho A_w U_w^2 + A_w \Delta p_w)$$

$$r A_d U_d 2 U_{\theta d} = r_w A_w U_w U_{\theta w}$$

$$A_\infty U_\infty = A_d U_d$$

$$A_d U_d = A_w U_w$$

$$\frac{1}{2}\rho U_\infty^2 = \frac{1}{2}\rho U_d^2 + \frac{1}{2}\rho U_{rd}^2 + p_{d+}$$

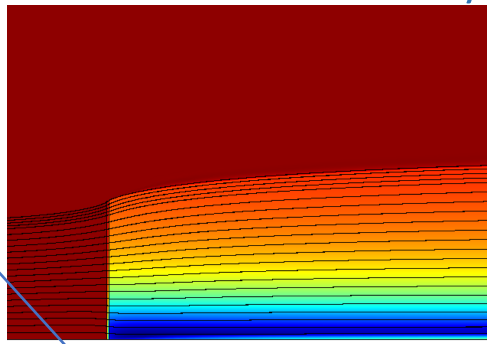
$$\tfrac{1}{2}\rho\left(U_d^2 + (2U_{\theta d})^2 + U_{rd}^2\right) + p_{d-} = \tfrac{1}{2}\rho\left(U_w^2 + U_{\theta w}^2\right) + \Delta p_w$$

$$F_x = A_d(p_{d+} - p_{d-})$$

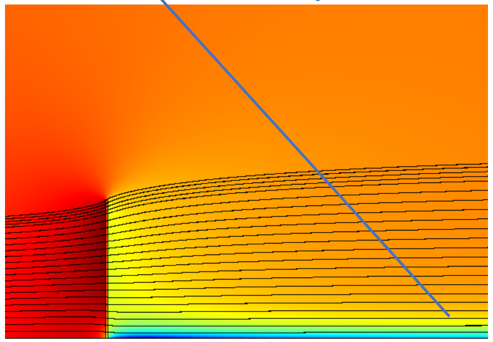
$$F_\theta = \rho A_d U_d 2 U_{\theta d}$$

$$\frac{F_x}{F_\theta} = \frac{\lambda U_\infty \frac{r}{R} + U_{\theta d}}{U_d}$$

Wake swirl velocity



Wake pressure



and why? - the founding fathers of BEM did not have  *and lived in a world without*  *, so a few % error was ok ..*

Far-wake pressure drop
Due to swirl (relatively easy …)

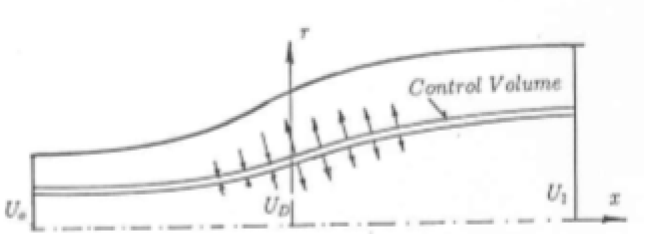
$$\Delta p_w(r) = \int_{R_w}^{r} \rho \frac{U_{\theta w}^2(\tilde{r})}{\tilde{r}} d\tilde{r}$$

Angular momentum enforcement
(relatively easy …)

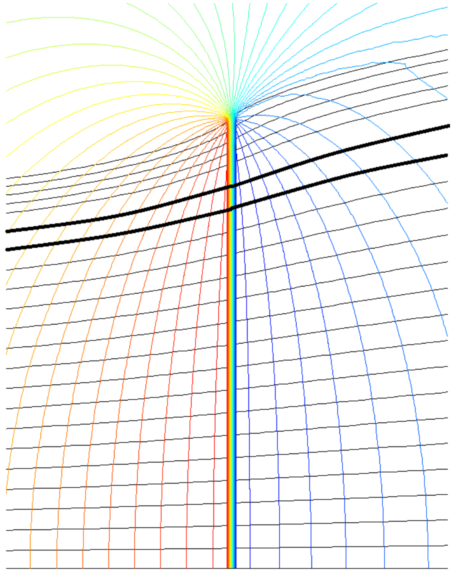
$$r A_d U_\mathrm{d} 2 U_{\theta\mathrm{d}} = r_w A_w U_\mathrm{w} U_{\theta\mathrm{w}}$$

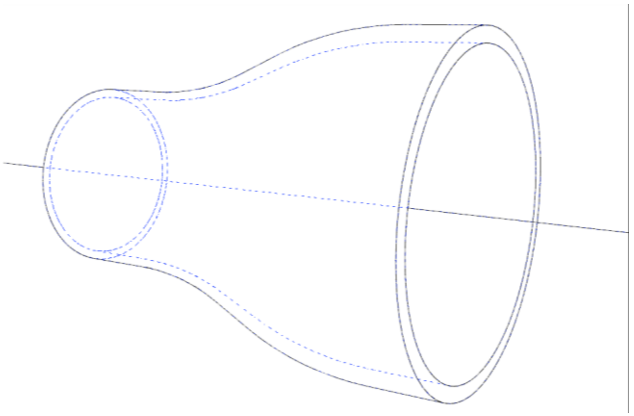
Axial projection of force
Interaction between
adjacent stream tubes
(not that easy)

$$\Delta F_s = \Delta C_{ts} \frac{1}{2} \rho A_d U_\infty^2 \quad , \quad \Delta C_{ts} = \frac{2\pi}{\frac{1}{2}\rho A_d U_\infty^2}\left(\int_{-\infty}^{\infty} p\bar{\boldsymbol{n}} \cdot \bar{\boldsymbol{n}}_x r_{in} dx - \int_{-\infty}^{\infty} p\bar{\boldsymbol{n}} \cdot \bar{\boldsymbol{n}}_x r_{out} dx \right)$$



From R. Mikkelsen, 2001





Corrected BEM results and benchmarks

Figure 1: Actuator Disk ΔC_{ts} results:

0.04 - 0.05 in ΔC_{ts}
Streamline thrust
coefficient

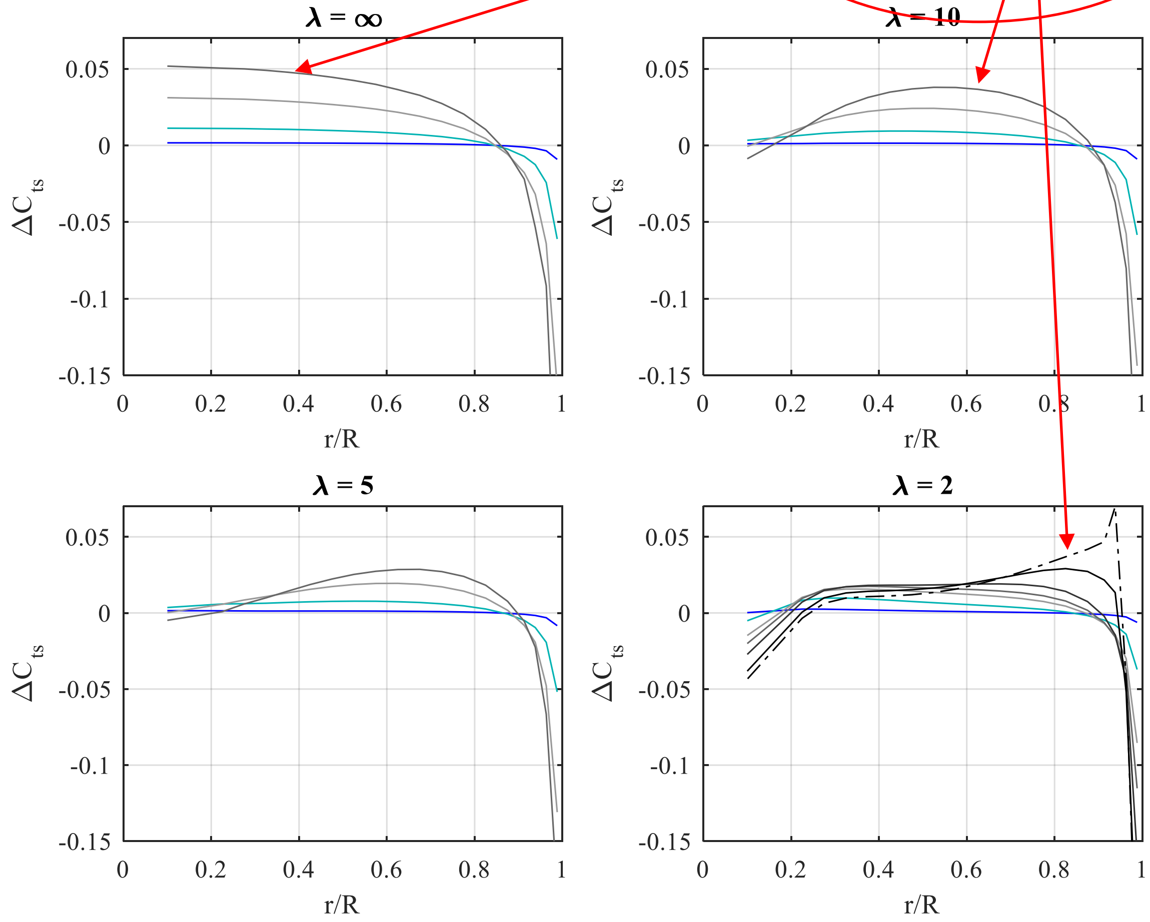
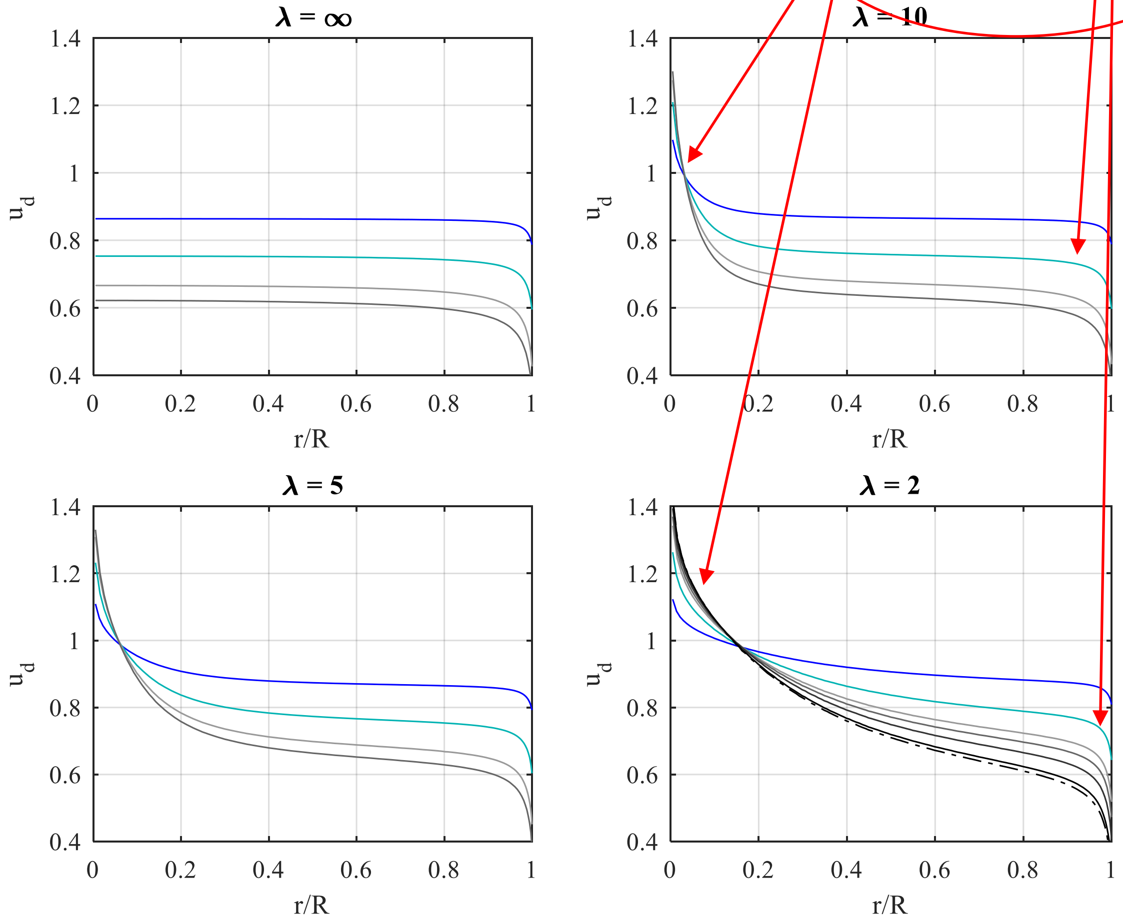


Figure 2: Actuator Disk **BENCHMARK** results: u_d , for different levels of rotor loading and tip-speed ratio λ.

Inboard speed-up
Tip speed-down



Corrected BEM results and benchmarks

3-5 % error in flow velocity at normal loading

Error reduced significantly !!

Figure 3: Classical BEM error (Actuator Disk reference)

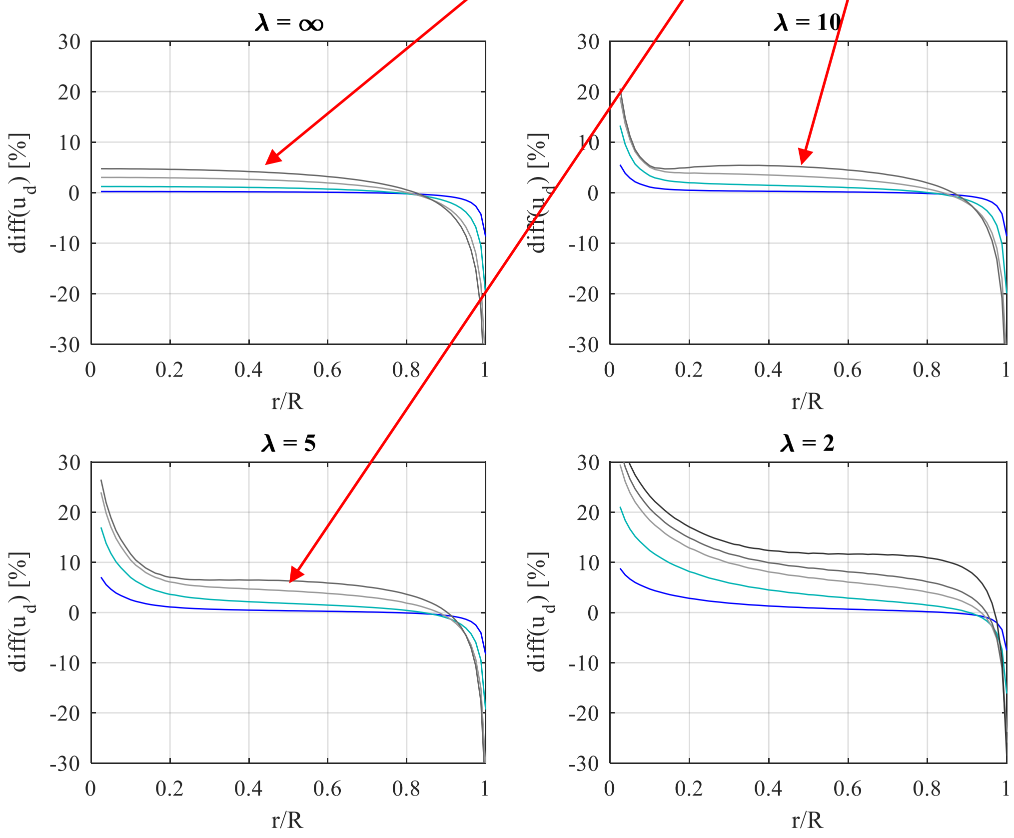
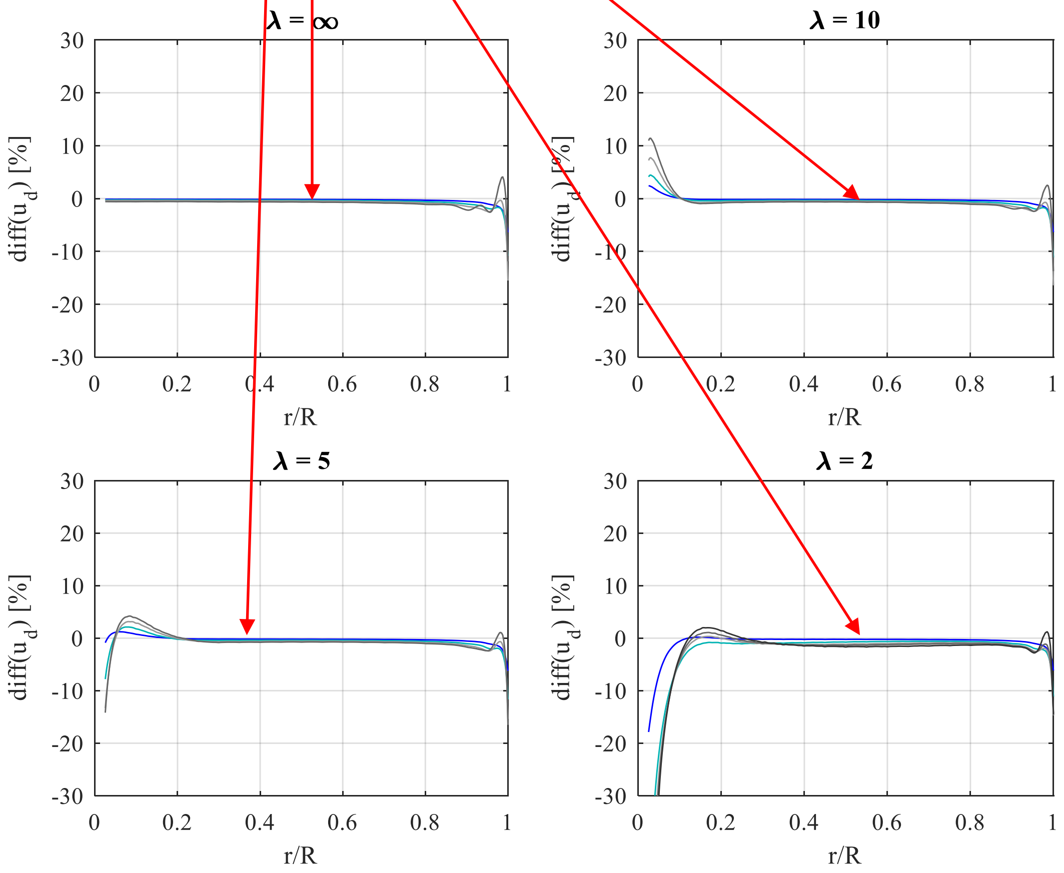


Figure 4: Corrected BEM error (Actuator Disk reference)



Spot on power prediction with Corrected BEM !

Standard BEM rotor power quite accurate for high TSR

Figure 5: Optimal power along blade: Classical BEM, corrected BEM, and Actuator Disk ref.

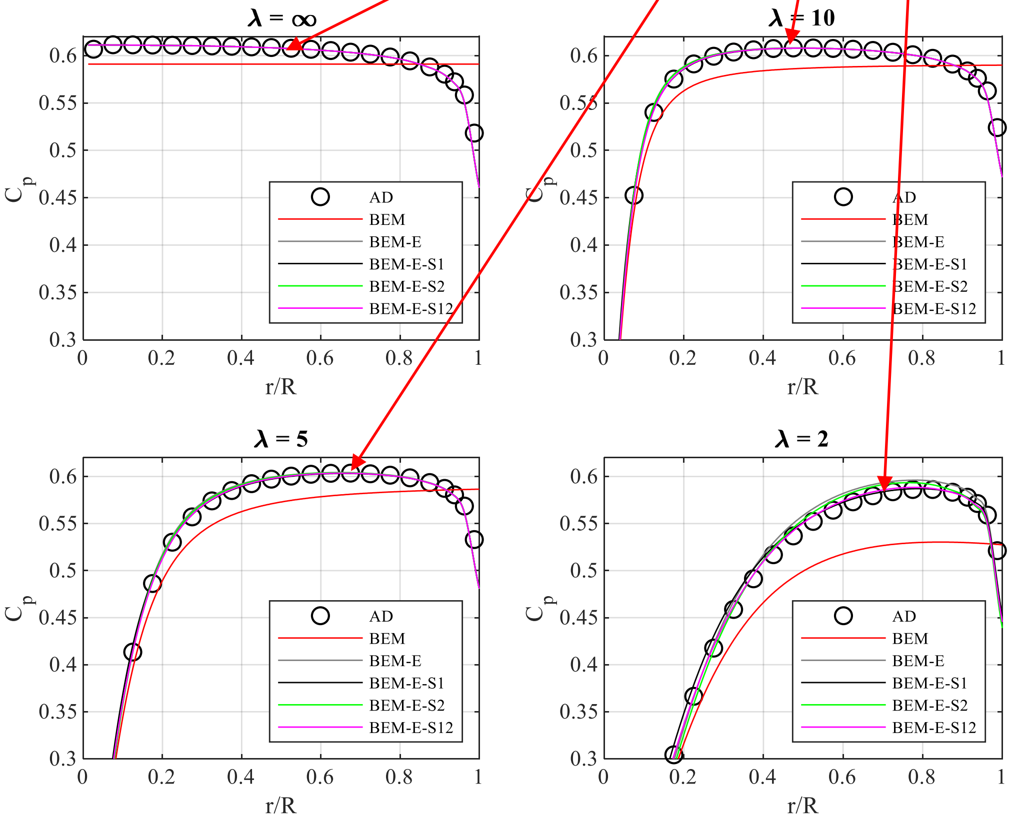
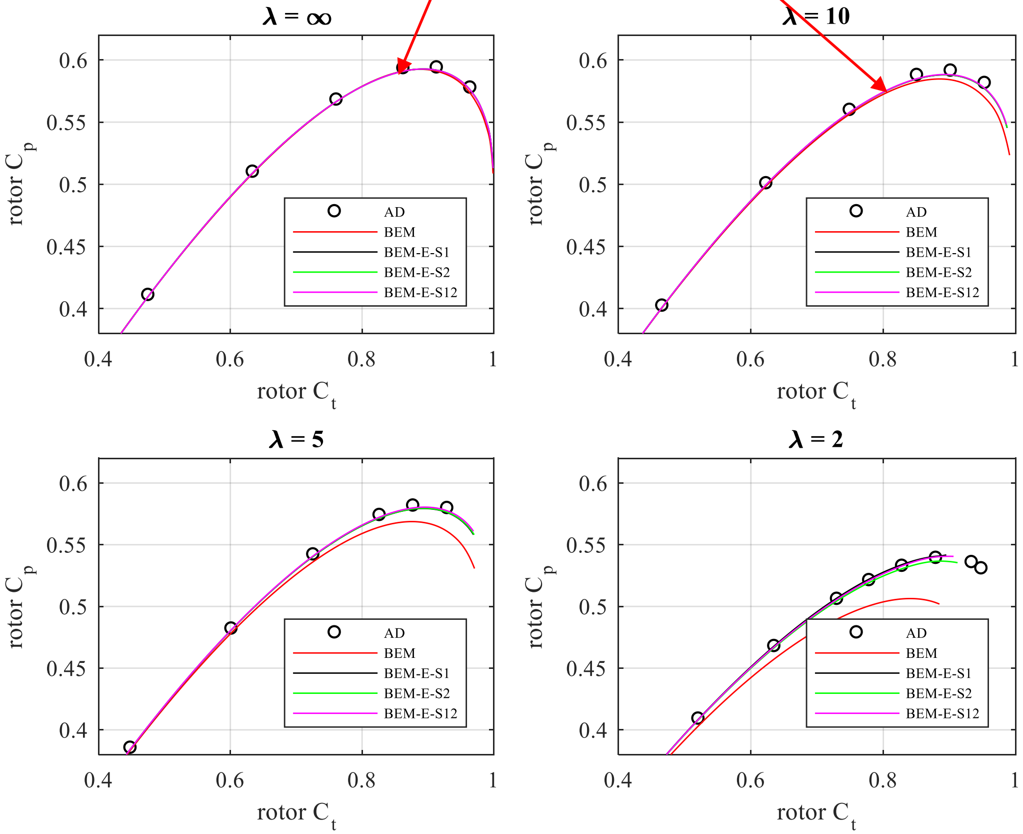


Figure 6: Optimal rotor power: Classical BEM, corrected BEM, and Actuator Disk ref.



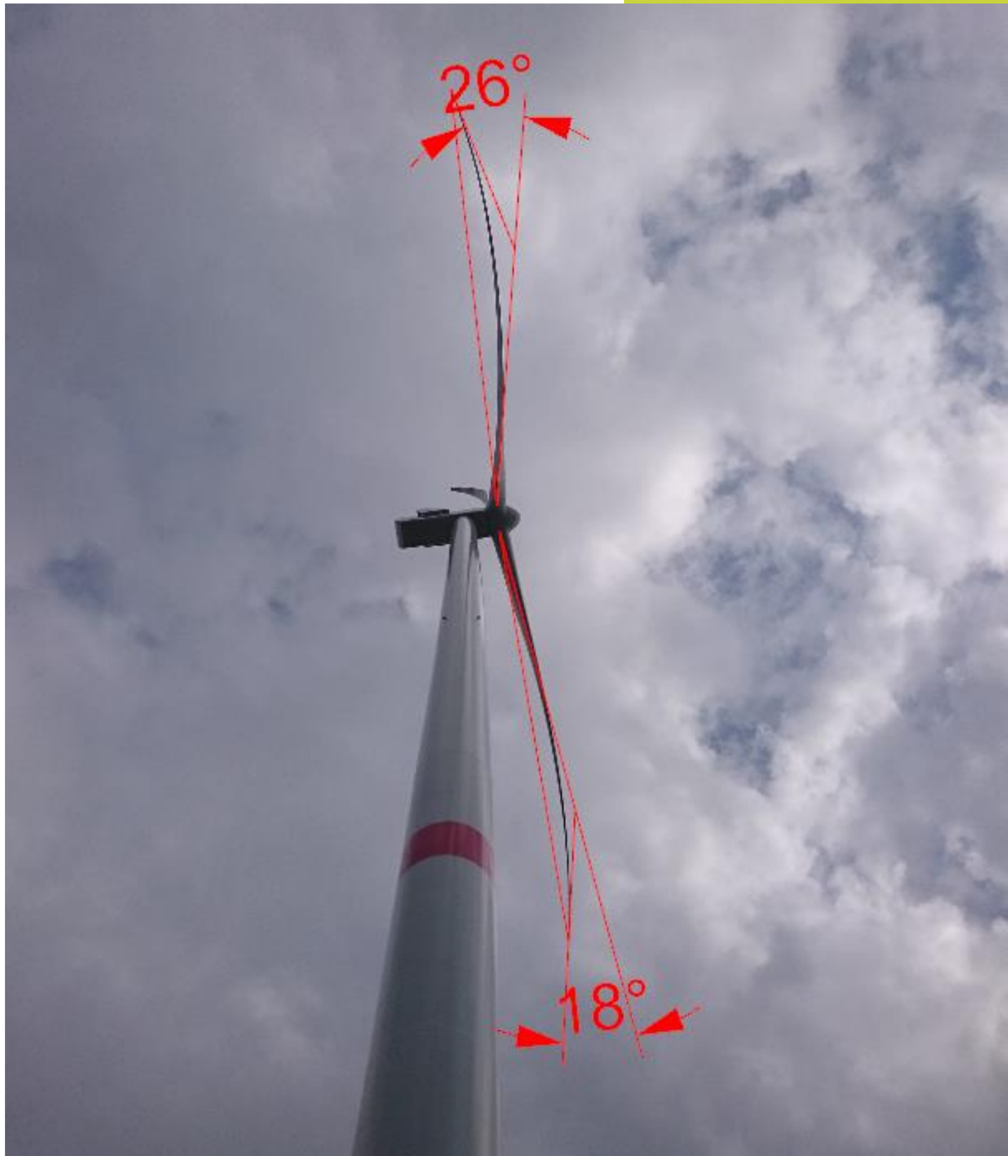
- ✓ **ACOMPLISHED**: The corrected BEM provides the missing terms to the classical BEM and imposes strict conservation of axial and angular momentum.
- ✓ **VALIDATION**: The improved accuracy is evidenced by match with Actuator Disk reference.
- ✓ **STANDARD BEM ERROR ESTIMATION**: This improved accuracy leads to elimination of a systematic misprediction of the classical BEM:
  - 3-5% rotor passage velocity, equivalent to 0.25-0.50 deg error on Angle-of-Attack
  - Higher AoA errors towards blade tip and blade root.
- ✓ **STANDARD BEM ERROR IMPORTANCE:** 0.25-0.50 AoA-error is not negligible but considered permissible on wind turbines.
- ✓ **NEED FOR CORRECTED BEM**: The corrected BEM is therefore “nice-to-have”, not “need-to-have”.
- ✓ **FUTURE**: Future need for narrower design tolerances and higher predictive accuracy might lead to increased need for the corrected BEM.

The mapping of the ΔF_s term by using CFD **<u>has been used to</u>**:

- Close the axial momentum gap in the standard BEM for **<u>in-plane</u>** rotors.

Mapping of the same ΔF_s term by using CFD **<u>could also be used to</u>**:

- Close the axial momentum gap in the standard BEM for **<u>out-of-plane</u>** rotors. Out-of-plane blade deflections are increasingly relevant for future turbines. AoA-errors from large out-of-plane deflections during normal operation would **<u>amplify</u>** the tendencies already identified for the plane rotor, i.e.:
  - Reveal further increased AoA on inner/mid part of rotor.
  - Reveal further decreased AoA on outer part of rotor.
  - Combined AoA-error will therefore increase on both inner/mid and outboard part of the blade



Thank you!